Integrerend project systeemtheorie

21/01/2013, Monday, 9:00-12:00

1 $(4+4+8+4=20)$ **Linearization**

Consider the so-called Van der Pol system

$$\ddot{z}(t) - \mu(1 - z^2(t))\dot{z}(t) + z(t) = 0.$$

(a) Write the system in the form of a nonlinear state-space system ($\dot{x} = f(x)$) by taking $x_1(t) = z(t)$ and $x_2(t) = \dot{z}(t)$.

(b) Show that $x_1(t) = x_2(t) = 0$ is a solution of $\dot{x} = f(x)$.

(c) Determine the linearized system.

(d) For which values of μ is the linearized system asymptotically stable.

2 (15) **Routh criterion**

Consider the linear system

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -4 & -a \end{bmatrix} x$$

where a is real number. For which values of a is this system asymptotically stable?

3 $(3+4+4+4+4+4+8+8=35)$ **Controllability and observability**

Consider the linear system

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad y = \begin{bmatrix} 0 & 1 \end{bmatrix} x.$$

Explain your answers to the following questions:

(a) Is it stable?

(b) Is it controllable?

(c) Is it observable?

(d) Is it stabilizable?

(e) Is it detectable?

(f) Does there exist an observer of the form $\dot{\hat{x}} = P\hat{x} + Qu + Ry$?

(g) Does there exist a stabilizing dynamic compensator (from y to u)? If yes, determine such a compensator.

4 $(3+5+12=20)$ **Detectability**

Consider the linear systems

$$\begin{aligned} \dot{x}(t) &= Ax(t) \quad x(0) = x_0 \\ y(t) &= Cx(t) \end{aligned}$$

where $x \in \mathbb{R}^n$ is the state and $y \in \mathbb{R}^m$ is the output. Let $x(t, x_0)$ denote the state trajectory of the system corresponding to the initial condition x_0. Define

$$W = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

and

$$\mathcal{V} = \{x_0 \mid \lim_{t\to\infty} x(t, x_0) = 0\}.$$

Show that

(a) $\mathcal{V}$ is a subspace.

(b) $\mathcal{V}$ is A-invariant.

(c) if $\ker W \subseteq \mathcal{V}$ then the system is detectable.

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10 pts gratis.